A Multi-Criteria Method for Noise Reduction

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Abstract

The article presents two multi-criteria methods for noise reduction. The idea of these methods based on a combination of two criteria. The first criteria is the sum of squares of the finite differences of the first and/or the second order. The second criterion is square difference of input signal and its evaluation. In the work, we prove the convergence and uniqueness of solutions obtained. We discuss influence of method's parameters on the result of noise reduction. On the set of test signals, we show the effectiveness of smoothing and noise reduction.

1. Introduction

In modern electronic systems during transmission the signal suffers of additive noise. The process of receive and conversion of the signal to digital form is associated with addition of noise component. Particular parameters of such a noise associated to change of temperature, change of electrical components parameters, interference with other radio-sources, possible impurities in sensitive elements and other issues. In most cases, the noise is additive. The main task of digital signal processing is to extract the useful signal and suppress the noise component. In the case of rapid signal changes and excesses in the useful signal the problem of signal denoising become more complicated. The preservation of these areas and noise reduction will be the subject of research. For this we use the multi-criteria approach based on the simultaneous minimization of two criteria.

Over the past 50 years, represented a large number of signal processing techniques [1-7]. The effectiveness of methods depends on the availability of information about the signal and noise. In the case of her lack of task becomes difficult.

The relevance of developing new method is based on the necessity of flexibility in adapting the devices for a wide range of tasks. An example of this problem is the ECG data processing [8, 9]. Development of filter allows you to simultaneously allocate isoline, denoising and keep of peaks. This is very urgent task in mobile devices.

The goal of this research is to the development of the method and its adaptation for the filter with opportunity to simultaneously allocate isoline, denoising and keep of peaks. This method finds implementation in mobile devices.

2. Model of input signal

A simplified discrete mathematical model of input signal $y(t_k) = y_k$ could be represented as the sum of desired signal model $s(t_k) = s_k$ and additive noise $N(t_k) = N_k$:

$$y(t_k) = s(t_k) + N(t_k),\ k = \overline{1, n}.$$

The functional dependence of the desired signal and the law of distribution of additive noise are unknown a priori. As a model of the noise component zero expectation and Gaussian distribution law, we use $N(m = 0)$.

As criterion of efficiency we apply the mean squared error (MSE). The estimation of the efficiency of the results of processing are carried out on a test set of signals of different forms with random additive noise. The set contains 100000 different implementations of processed signal.

3. Method of noise reduction

3.1 Objective function with employing two criteria.

In the work to reduce the noise variance, we will decrease the sum of the squares of the finite-difference of its values:

$$\sum_{k=1}^{n-1}\left(\bar{s}_k-\bar{s}_{k+1}\right)^2, \qquad (1)$$

As well as to minimize of the sum of squares of finite differences of the second order:

$$\sum_{k=1}^{n-2}\left(\bar{s}_k-2\cdot\bar{s}_{k+1}-\bar{s}_{k+2}\right)^2. \qquad (2)$$

At the same time as a measure of discrepancy between the input and desired signal is used:

$$\sum_{k=1}^{n}\left(\bar{s}_k-y_k\right)^2. \qquad (3)$$

The result of the smoothing is obtained and show to reduce the sum of (1 and (or) 2) and (3). This is obtained by minimization of objective function employing two criteria of form:

$$\varphi(\bar{s}_1,\bar{s}_2,...,\bar{s}_n)=\alpha\sum_{k=1}^{n}(\bar{s}_k-y_k)^2+\sum_{k=1}^{n-1}(\bar{s}_k-\bar{s}_{k+1})^2, \qquad (4)$$

$$\varphi(\bar{s}_1,\bar{s}_2,...,\bar{s}_n)=\alpha\sum_{k=1}^{n}(\bar{s}_k-y_k)^2+\sum_{k=1}^{n-2}\left(\bar{s}_k-2\cdot\bar{s}_{k+1}-\bar{s}_{k+2}\right)^2, \qquad (5)$$

and minimization of the multi-criteria objective function of form:

$$\varphi(\bar{s}_1,\bar{s}_2,...,\bar{s}_n)=\alpha\sum_{k=1}^{n}(\bar{s}_k-y_k)^2+\beta\sum_{k=1}^{n-1}(\bar{s}_k-\bar{s}_{k+1})^2+\sum_{k=1}^{n-2}\left(\bar{s}_k-2\cdot\bar{s}_{k+1}-\bar{s}_{k+2}\right)^2, \qquad (6)$$

here α and β – constant adjustment factors.

We implement these methods of noise reduction and have observed that the best results based on the use of simulation are reached for $0{,}01\le\alpha\le 4.44$, in the case of expressions (4) and (5), and $0{,}01\le\alpha\le 10$, $0{,}01\le\beta\le 10$ in the case of (6).

Proposed functions (4-6) are continuous and limited below by a set of R^n, and at least in one point $(\bar{s}_1,\bar{s}_2,...,\bar{s}_n)$ reaches its minimum value. We prove the uniqueness of this point, the example of the target function of the form (4). Due to the necessary conditions of an extremum, its coordinates must satisfy the system of equations:

$$\frac{\partial\varphi}{\partial X_j}=0,\quad j=\overline{1,n}, \qquad (7)$$

i.e. the following system of n linear equations with n unknowns $\bar{s}_1,\bar{s}_2,...,\bar{s}_n$:

$$\begin{cases}(1+\alpha)\bar{s}_1-\bar{s}_2-\alpha y_1=0;\\(2+\alpha)\bar{s}_k-\bar{s}_{k+1}-\bar{s}_{k-1}-\alpha y_k=0, \qquad k=2,3,...,n-1;\\(1+\alpha)\bar{s}_n-\bar{s}_{n-1}-\alpha y_n=0.\end{cases} \qquad (8)$$

or:

$$\begin{cases}\bar{s}_2=(1+\alpha)\bar{s}_1-\alpha y_1;\\\bar{s}_{k+1}=(2+\alpha)\bar{s}_k-\bar{s}_{k-1}-\alpha y_k, \qquad k=2,3,...,n-1;\\(1+\alpha)\bar{s}_n-\bar{s}_{n-1}-\alpha y_n=0.\end{cases} \qquad (9)$$

We have proved that the system of equations (9) has a unique solution. We have applied the method of mathematic induction and check the validity of the approval P_k : «The first $(k-1)$ equation of the system (9) make it possible to define variables $\bar{s}_1,\bar{s}_2,...,\bar{s}_k$, as linear functions of the argument $\bar{s}_1$, i.e. $\bar{s}_j=\alpha_j\bar{s}_1+\beta_j$, when $\alpha_{j-1}<\alpha_j$, $j=\overline{1,k}$» for all $k=\overline{1,n}$ (for accept $\alpha_0=0$). For $k=1$ get $\alpha_1=1\,(>0)$, $\beta_1=0$, and in the case $k=2$ – $\bar{s}_2=\alpha_2\bar{s}_1+\beta_2$, when $\alpha_2=1+\alpha>\alpha_1$, $\beta_2=-\alpha Y_1$, i.e. mathematical statements P_1, P_2 are true. Assuming for approval of fidelity P_k for all $2\le k<n$, for the prove P_{k+1}. From the equation k of (9) we obtain

$$\bar{s}_{k+1}=(2+\alpha)(\alpha_k\bar{s}_1+\beta_k)-(\alpha_{k-1}\bar{s}_1+\beta_{k-1})-\alpha Y_k=\alpha_{k+1}\bar{s}_1+\beta_{k+1},$$

here $\alpha_{k+1}=(2+\alpha)\alpha_k-\alpha_{k-1}>\alpha_k$; $\beta_{k+1}=(2+\alpha)\beta_k-\beta_{k-1}-\alpha y_k$.

The approval $P_1,P_2,\dots,P_n$ is confirmed. Use the mathematical statements P_n the last equation in (9) takes the form $\gamma\bar{s}_1+\delta=0$, when $\gamma=(1+\alpha)\alpha_n-\alpha_{n-1}=(\alpha_n-\alpha_{n-1})+\alpha\cdot\alpha_n>0$, $\delta=(1+\alpha)\beta_n-\beta_{n-1}-\alpha Y_n$. The resulting equation has a unique solution $\bar{s}_1 = -\frac{\delta}{\gamma}$. It is uniquely determined by the values $\bar{s}_k = \alpha_k \bar{s}_1 + \beta_k$, when $k = \overline{2, n}$.

Thus, the system of equations (4) has a unique solution. Similarly we can prove uniqueness of the solutions for the objective functions of the form (5) and (6).

To find the point of the smallest the objective function $\varphi(\bar{s}_1, \bar{s}_2, ..., \bar{s}_n)$ value (4), (5) and (6) use the method of steepest descent [10]. We establish accuracy $\varepsilon > 0$ with which to be found the values $\bar{s}_1, \bar{s}_2, ..., \bar{s}_n$. As a first iteration, we will $\bar{s}_k = y_k$, $k = \overline{1, n}$.

3.2 Minimization method for smoothing the noise**.**

For all $1 \le k \le n$ we set the value a_k. For this we get it with the left side of the k-th equation of system (9).

For the objective function (5):

$$\begin{cases} (1+\alpha)\cdot\bar{s}_1 - 2\cdot\bar{s}_2 + \bar{s}_3 - \alpha\cdot y_1 = 0; \\ (5+\alpha)\cdot\bar{s}_2 - 2\cdot\bar{s}_1 - 4\cdot\bar{s}_3 + \bar{s}_4 - \alpha\cdot y_2 = 0; \\ \bar{s}_{k-2} - 4\cdot(\bar{s}_{k-1} + \bar{s}_{k+1}) + (\alpha+6)\cdot\bar{s}_k + \bar{s}_{k+2} - \alpha\cdot y_k = 0, \quad 2 < k < n-2; \\ \bar{s}_{n-3} - 4\cdot\bar{s}_{n-2} + (\alpha+5)\cdot\bar{s}_{n-1} + 2\cdot\bar{s}_n - \alpha\cdot y_{n-1} = 0; \\ \bar{s}_{n-2} - 2\cdot\bar{s}_{n-1} + (\alpha+1)\cdot\bar{s}_n - \alpha\cdot y_n = 0. \end{cases} \quad (10)$$

For the objective function (6), reduces to the solution of the system:

$$\begin{cases} (1+\alpha+\beta)\cdot\bar{s}_1 - (2+\beta)\cdot\bar{s}_2 + \bar{s}_3 - \alpha\cdot y_1 = 0; \\ (5+\alpha+2\cdot\beta)\cdot\bar{s}_2 - (\beta+2)\cdot\bar{s}_1 - (\beta+4)\cdot\bar{s}_3 + \bar{s}_4 - \alpha\cdot y_2 = 0; \\ \bar{s}_{k-2} - (\beta+4)\cdot(\bar{s}_{k-1} + \bar{s}_{k+1}) + (\alpha+2\cdot\beta+6)\cdot\bar{s}_k + \bar{s}_{k+2} - \alpha\cdot y_k = 0, \quad 2 < k < n-2; \\ \bar{s}_{n-3} - (\beta+4)\cdot\bar{s}_{n-2} + (\alpha+2\cdot\beta+5)\cdot\bar{s}_{n-1} - (\beta-2)\cdot\bar{s}_n - \alpha\cdot y_{n-1} = 0; \\ \bar{s}_{n-2} - (\beta+2)\cdot\bar{s}_{n-1} + (\alpha+\beta+1)\cdot\bar{s}_n - \alpha\cdot y_n = 0. \end{cases} \quad (11)$$

For the objective function (4) we set:

$$q = \alpha \sum_{k=1}^{n} a_k^2 + \sum_{k=1}^{n-1} (a_k - a_{k+1})^2 \,. \quad (12)$$

For the objective function (5):

$$q = \alpha \cdot \sum_{i=1}^{n} a_i^2 + \sum_{i=1}^{n-2} (a_i - 2\cdot a_{i+1} + a_{i+2})^2 \,. \quad (13)$$

For the objective function (6.7):

$$q = \alpha \cdot \sum_{i=1}^{n} a_i^2 + \beta \cdot \sum_{i=1}^{n-1} (a_i - \bar{s}_{i+1})^2 + \sum_{i=1}^{n-2} (a_i - 2\cdot a_{i+1} + a_{i+2})^2 \,. \quad (14)$$

If $q = 0$, in the point $\bar{s} = (\bar{s}_1, \bar{s}_2, ..., \bar{s}_n)$ the function φ reaching the lowest value. Note that $\bar{a} = (a_1, a_2, ..., a_n) = \frac{1}{2} \operatorname{grad} \varphi(\bar{s})$ and that $q = 0$ if and only if $\bar{a} = \bar{0}$. If value $q \ne 0$ the function $f(t) = \varphi(\bar{s} + t \cdot \bar{a})$ it is a quadratic function with a positive second derivative. Solving the equation $f'(t) = 0$, find the minimum point of the objective function of the form (4):

$$t = \frac{\alpha \sum_{k=1}^{n} a_k (y_k - \bar{s}_k) + \sum_{k=1}^{n-1} (a_{k+1} - a_k)(\bar{s}_k - \bar{s}_{k+1})}{q} \,, \quad (15)$$

for the objective function (5):

$$t = \frac{\alpha \cdot \sum_{i=1}^{n} a_i \cdot (y_i - \bar{s}_i) + \sum_{i=1}^{n-2} (2\cdot a_{i+1} - a_i + a_{i+2}) \cdot (\bar{s}_i - 2\cdot\bar{s}_{i+1} + \bar{s}_{i+2})}{q} \,, (16)$$

and for the objective function (6):

$$t = \frac{\alpha \cdot \sum_{i=1}^{n} a_i \cdot (y_i - \bar{s}_i) + \beta \cdot \sum_{i=1}^{n-1} (a_{i+1} - a_i) \cdot (\bar{s}_i - \bar{s}_{i+1})}{q} + \frac{\sum_{i=1}^{n-2} (2\cdot a_{i+1} - a_i + a_{i+2}) \cdot (\bar{s}_i - 2\cdot\bar{s}_{i+1} + \bar{s}_{i+2})}{q} \quad (17)$$

In the point $\bar{s}$ has function derivative φ and be the direction vector $\bar{a}$ is positive, then $f'(0) > 0$; and consequently $t \ne 0$. On next step performed correction is values $\bar{s}_1, \bar{s}_2, ..., \bar{s}_n$:

$$\bar{s}_k = \bar{s}_k + t \cdot a_k \,, \; k = \overline{1, n} \,.$$

After this we check the mathematical expression

$$\max_{1 \le k \le n} |a_k| \le \frac{\varepsilon}{|t|\sqrt{n}} \,. \quad (18)$$

If inequality (18) holds, the required accuracy is considered to be achieved, and the calculation ends. Then $|t\bar{a}| = |t| \sqrt{\sum_{k=1}^{n} a_k^2} \le |t| \sqrt{n} \max_{1 \le k \le n} |a_k| \le \varepsilon$, ie the distance between the last two iterations in the space R^n does not exceed of the values ε. Upon failure to perform of condition (18) is repeated calculation $q, t, \bar{s}_1, \bar{s}_2, \ldots, \bar{s}_n$ and verification of the previous step.

The vector estimates $(\bar{s}_1, \bar{s}_2, ..., \bar{s}_n)$ iteratively corrected so that the objective function $\varphi(\bar{s}_1, \bar{s}_2, ..., \bar{s}_n)$ reached its lowest value. At some stage of the iterative process is performed expression (18) and calculating terminated. The resulting vector estimates $(\bar{s}_1, \bar{s}_2, ..., \bar{s}_n)$ with accuracy ε will be minimum point of the objective function $\varphi(\bar{s}_1, \bar{s}_2, ..., \bar{s}_n)$ for given initial conditions [13].

4. The impact of the method parameters on the result

For digital signal processing in real time we propose use the objective function σ_{MSE} in the window k, with subsequent sliding l over all input implementation to finding evaluations for multi-criteria.

Choice of the value of the processing window the shown in Fig. 1,2 and is based on a minimum iteration cost, for the to get the result of processing the input signal to the $n > 20$, $k = 10$ [14].

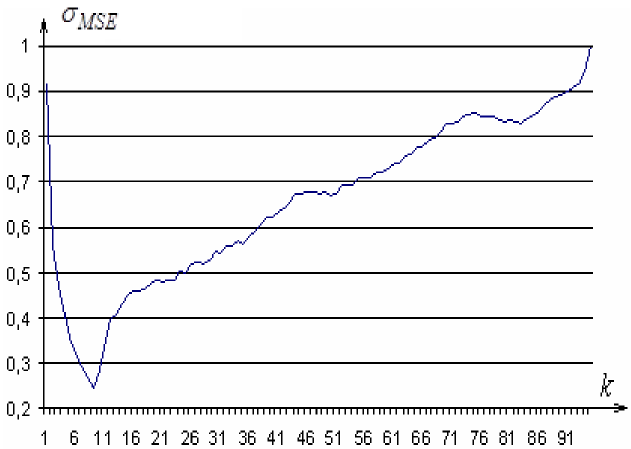

Figure. 1 Graph of standard deviation values from of the window width

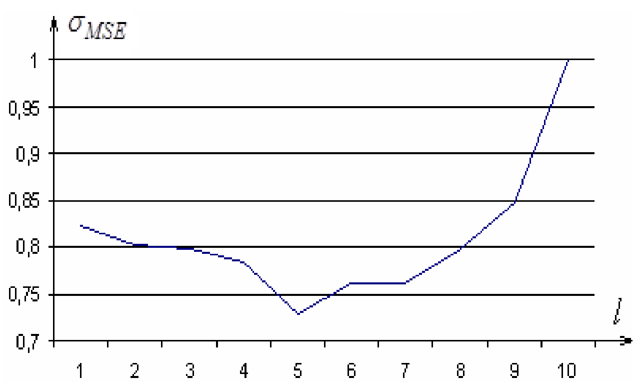

Figure. 2 Graph of standard deviation values from of the step size of the sliding window

Analysis of the results presented in Fig. 1 has shown that, minimum estimation $\sigma_{MSE}(k)$ achieved with $k = 10$, and $\sigma_{MSE}(l)$ achieved with $l = 5$ that weakly dependent on the function of the useful component of the S_k Fig. 2 shows the dependences of $\sigma_{MSE} = f(\alpha)$. The values obtained by smoothing of starting implementations of multi-criteria objective function (5). The starting signals S_k is are described: the united model (curve 1), the triangular shape (curve 2), the exponential function (line 3), the parabolic function (curve 4) and the harmonic form (line 5). In the same time we use additive noise with a Gaussian distribution law and variance σ_{noise} =0.01, ei $N_k \in N(0,0.1)$ [13].

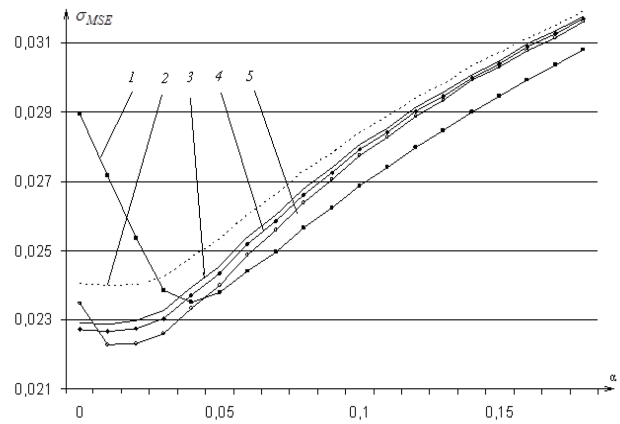

Figure. 3 Graph parameter changes α depending on the shape of the signal

Analysis of the results presented in Fig. 3 shows that the use of two-criteria of the objective function (5) allows to localize the value α in one segment $0{,}01 \le \alpha \le 0{,}04$ (tab. 1). The table shows the data processing of the input signals of different forms s_k. Inaccuracy in selecting parameter α, increases the value to σ_{MSE} on 10% .

Table. 1 shows the values of α_{min} setting in which values are the minimum mean square error $\sigma_{MSE} = f(\alpha)$ [11].

Table 1. The values of α_{min} which MSE $\sigma_{MSE} = f(\alpha)$.

	α_{min1}	$\sigma_{MSE1} = f(\alpha)$	α_{min2}	$\sigma_{MSE2} = f(\alpha)$
the united model	0,04	0,023502	0,21	0,025858
the triangular shape	0,02	0,023961	0,08	0,026778
the exponential function	0,01	0,022876	0,08	0,032578
the parabolic function	0,01	0,022665	0,09	0,03423
the harmonic form	0,01	0,022271	0,21	0,041156

5. Signal processing as they become available

The process of obtaining estimates in a sliding window $\bar{s}_k$, carried out with parallel processing of initial values in a sliding window, with use of multi-criteria objective function with different processing parameters α. The rule of selecting α, is presented in peppers [10, 11].

The transition between estimates of the obtained with different parameters α, on the condition:

$$\bar{s}_k = \begin{cases} \bar{s}_k(\alpha_1) & (\bar{s}_k(\alpha_1) - \bar{s}_k(\alpha_2))^2 \le p \\ \bar{s}_k(\alpha_2) & (\bar{s}_k(\alpha_1) - \bar{s}_k(\alpha_2))^2 > p \end{cases}$$

when: $\bar{s}_k(\alpha_1)$, $\bar{s}_k(\alpha_2)$ – the result of processing with parameters $\alpha_1(\sigma_{MSE})$ and $\alpha_2(\sigma_{MSE})$, p – threshold which is defined experimentally and with dispersion of the additive noise $\sigma_{noise} < 0{,}2$ is $p = 0{,}1$.

Fig. 4 shows an example of the result of digital signal processing y_k (curve 1) represented in the form of an additive mixture of the useful signal (curve 2) and the noise component in the presence of impulse noise [13].

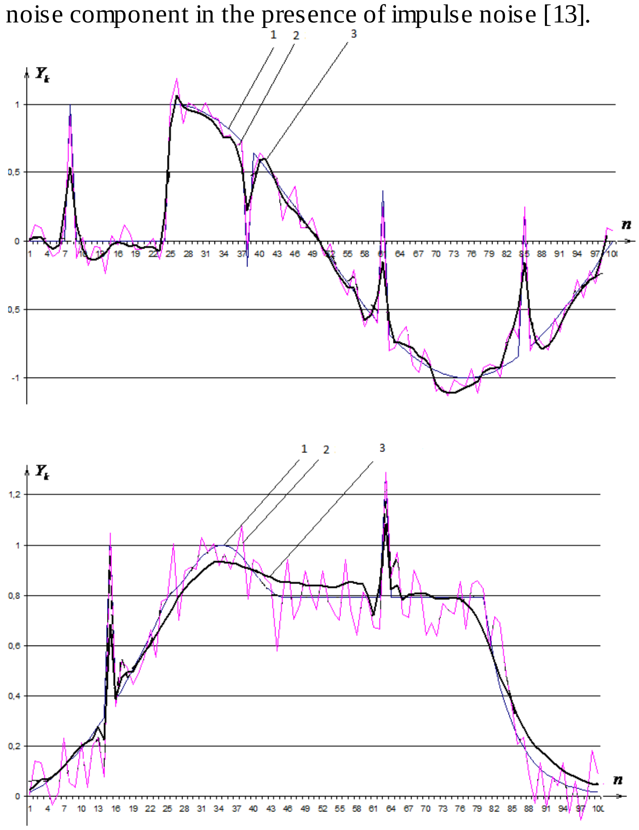

Figure. 4 Example of a digital signal smoothing in the presence of impulse noise

area of application of of the proposed method is to process ECG data. An example of the suppression of the noise component on the ECG and the allocation of isoline is shown in fig. 5.

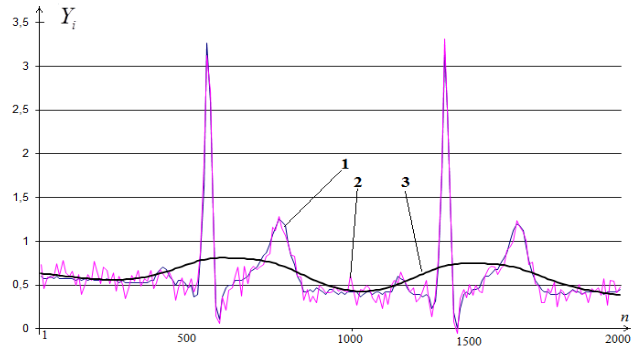

Figure. 5 An example of the definition of isoline on the ECG (1 - the result of processing, 2 – ECG with noise, 3 – the isoline).

6. Conclusion

In the paper, the multi-criteria methods of noise reduction the digital signal in a limited volume of a priori information about functions signal and the statistical characteristics of the noise are developed and studied.

Use a multi-criteria methods smoothing for the digital signal processing in a sliding window, gives the increased the efficiency by an average of 25% in comparison with the processing of the entire implementation. In the presence of in input implement the functions of discontinuity of the first kind or impulse noise, there is an increase in efficiency of 60% on average.

When analyzing the data ECG, the proposed method can effectively determine the the signal of heart rate and the isoline

7. Acknowlegement

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